GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ELECTRICAL ENGINEERING

ECE 6272 FALL 2010

COMPUTER PROJECT #1

**SAMPLE SOLUTION**

# MY RESULTS FOR RCS DISTRIBUTIONS

My results are presented in the several figures that follow. They were generated using the simulations RCS10\_project.m and xcorrRCS10\_project.m included at the end of these notes.

shows one of the 50 scatterer targets, each scatterer of RCS = 1 m2, used for the first few experiments. The blue crosses denote the scatterer positions with the 5m by 10 m target area. This collection of scatterers might be appropriate for some man-made vehicles, whose RCS is typically dominated by a relatively small number of scattering centers, such as corners between metal surfaces, seams, relatively flat plates normal to the radar line of sight, *etc.*.



*Figure 1. The blue asterisks show the distribution of 50 scatterers of  
fixed RCS = 1 m2 each.*

Figure 2 shows the RCS data for this particular target. This was computed using exactly Eqn. (2.51) from the textbook, which is repeated here:



Note the random-looking variation with aspect angle as the radar “walks around” the target. The angle increment for this data was 0.5° (720 angles). The peak theoretically-possible RCS is 10log10(502) = 34 dBsm; this would occur at an angle where all of the individual scatterer echoes happened to add exactly in-phase with one another, which is *extremely* unlikely. If the relative phases were all random with respect to one another, it can be shown that the expected value of the composite RCS would be 50, which is 17 dBsm. Visually, the “mean” of this data is indeed close to 17 dBsm, and the peak is about 27 dBsm. Most of the data falls within a dynamic range of about 10 dB, but the total dynamic range in this plot is nearly 40 dB.



*Figure 2. RCS vs. aspect angle for the scatterer distribution of Figure 1, using an X band radar at a nominal range of 10 km.*

Figure 3 is the normalized histogram of the data in Figure 2, using 100 histogram bins. Overlaid is the exponential pdf with the same mean RCS. (Note that the mean of the RCS data must be computed on a linear, not decibel, scale.) I normalized the histogram to have unit area as described in the problem assignment, and then plotted the theoretical curve for the pdf with no normalization, since it has unit area already by definition. The shape and spread of the histogram generally follow the theoretical curve, suggesting that it is a reasonable model, but the fit is somewhat ragged. Although not shown here, if the RCS is computed at a denser set of aspect angles (e.g., 0.2° or 0.1° spacing), the match of the histogram to the pdf curve tends to improve noticeably.

Figure 4 gives the result of averaging the histograms of ten different 50-scatterer targets; Figure 3 is the histogram of the first of these ten targets. Each of the individual histograms is similar in character to that of Fig. 3, but the details vary. However, the average of the 10 histograms provides a much smoother fit to the corresponding exponential pdf.



*Figure 3. Normalized histogram of data in Figure 2. Also shown is an exponential pdf with the same mean as the RCS data.*

Notice the high value of the ten-target histogram at the extreme right of Fig. 4. This occurs because I selected my bin boundaries for all of the histograms based on the data from the first target. As it happened, several of these particular random targets included higher values of RCS than did the first one. All of these values were placed into the highest RCS bin when their histograms were computed. Re-running my simulation will produce new and different random targets; sometimes there is a significant “bump” at the right end of the averaged histogram, sometimes not. A more reliable procedure would be to choose the bin boundaries based on the target data having the highest RCS value.

Figure 5 repeats this experiment using a fixed aspect angle, but stepping the RF frequency from 10 to 14.5 GHz in 300 15-MHz steps. The RCS data looks similar to that of Fig. 2. Fig. 5a is the normalized histogram with an overlaid exponential pdf having the same mean for a single target, while Fig. 5b is the average of the normalized histograms for ten targets, with an overlaid exponential pdf matching the mean of all the data. While the single-target histogram is even more ragged than in the angle-stepped case, it still tends to follow the exponential pdf shape, and averaging over multiple targets again significantly improves the fit to the exponential pdf.



*Figure 4. Average of the histograms of ten 50-scatterer targets. Also shown is an exponential pdf with the same mean as that of all of the RCS data from all ten targets.*

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |

*Figure 5. (a) Histogram of the RCS-vs.-frequency data for the target of Fig. 1 with overlaid exponential pdf. (b) Average of the RCS-vs.-frequency histograms for 10 targets with overlaid exponential pdf.*

# MY RESULTS FOR COMPLEX ECHO CORRELATION

Figure 6a shows the central portion of the magnitude of the complex autocorrelation function of the *complex* echo for another single 50-scatterer example (but not the same specific example as used earlier; this data is from a new sample target). The RCS data used for the autocorrelation was computed at 0.02° aspect angle increments around a nominal aspect angle of zero degrees, meaning we are looking head-on at the 5 m wide side of the target. Clearly, there is a well-defined and narrow correlation peak; however, it has significant and highly oscillatory sidelobes. The vertical lines shows the decorrelation interval predicted by the theoretical line array result *c*/2*LF* = 0.172°. Figure 6b expands the central peak. It is not obvious what to consider the decorrelation interval. The 1/*e* point (a value of 0.368) occurs at a little less than 0.13°, the point at which if first drops to the level of the first sidelobe is also about 0.13°, the first minimum occurs right at 0.18°, which is in nice agreement with the prediction of the first zero. On the other hand, the first minimum isn’t really much of a zero.

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |

*Figure 6. Autocorrelation in angle of a single 50-scatterer target viewed from the 10 m wide side (a) Magnitude of autocorrelation function.. Vertical lines indicate theoretical decorrelation point. (b) Expansion of central peak of (a).*

Figure 7 shows the result obtained from averaging this autocorrelation with the autocorrelations of 9 similar targets. Note that it is the complex autocorrelation function that is averaged; the magnitude is not taken until after the averaging, and then only for display purposes. The autocorrelation magnitude outside of the mainlobe is generally significantly lower now. The first minimum, which is now much closer to zero, is at about 0.2° in this case, a little larger than our prediction, but not bad.

(By the way, if you took the magnitude before the averaging, your averaged autocorrelation probably had a significant triangular base to it; think about why this happened by considering the magnitude to be composed of a constant “DC” value plus the variable part of the magnitude of a correlation function.)

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |

*Figure 7. Average of angle autocorrelation functions of ten 50-scatterer target viewed from the 5 m wide side (a) Magnitude of autocorrelation. Vertical lines indicate theoretical decorrelation point. (b) Expansion of central peak of (a).*

Figure 8 shows the results for decorrelation in frequency for a fixed aspect angle of 0°. Viewed from this angle, the length of the target along the radar line of sight (LOS) is 10 m; this is the value of the term *L*sin** in the formula for predicting the decorrelation angle in frequency. Specifically, the predicted value is *F* = *c*/2(10)= 15 MHz. For this data, the first local minimum of the 10-target autocorrelation function seems to be a natural choice for defining the decorrelation interval. Under this standard, the observed decorrelation interval in frequency is 14.9 MHz, a very good match to the predicted 15 MHz.

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |

*Figure 8. Autocorrelation in frequency of 50-scatterer target viewed from 0° aspect angle, so that the length along the radar line of sight is 10 m. (a) Magnitude of autocorrelation function. Vertical lines indicate theoretical decorrelation point. (b) Expansion of central peak of (a).*

# MY MATLAB CODE

Listing of RCS10\_project.m

% RCS10\_project

% Does a simple-minded relative RCS polar plot for a "complex

% target" consisting of a random collection of point scatterers.

% No multiple bounce or any such nonsense.

%

% Mark Richards, September 2006

%

% Updated for averaging over multiple target, September 2010

clear all

close all

Nt=10; % # of targets to average over

% Set number and locations of scatterers. They are uniformly

% distributed within a box 5 m by 10 m centered at the origin.

% N=input('Enter number of scatterers to use: ');

N=50;

% each column of 'x' or 'y' are the scatterer corrdinates for a single

% target

y=5\*rand(N,Nt)-2.5; x=10\*rand(N,Nt)-5;

% plot the scatterer distribution for the first target as an example

figure(1)

plot(x(:,1),y(:,1),'\*');

xlabel('x'), ylabel('y')

title('Distribution of Scatterers')

axis('equal'); axis([-5,5,-2.5,2.5]);

% Amplitudes are fixed = 1

z=1;

% Input number of angles and nominal range and frequency

% M=input('Enter # of angles: ');

% R=input('Enter nominal range (m): ');

% f=input('Enter frequency (Hz): ');

M=round(360/0.5); R=10e3; f = 10e9;

% Loop over aspect angle to do complex voltage estimates

q=zeros(M); echo=zeros(M,Nt);

% Loop over radar-target aspect angles

for k=1:M

q(k)=(2\*pi/M)\*(k-1); % current aspect angle

% Loop over targets and individual point scatterers

% each column of 'echo' is a different target

for r = 1:Nt

for p=1:N

phasor=z\*exp(j\*4\*pi\*f\*norm([x(p,r)-R\*cos(q(k)),y(p,r)-R\*sin(q(k))])/3e8);

echo(k,r)=echo(k,r)+phasor;

end

end

end

% The magnitude-squared of the total complex echo

% is the RCS to within a constant

RCS=abs(echo).^2;

RCSdB=10\*log10(RCS);

% Plot the dB data for the first target as an example

figure(2)

plot((180/pi)\*q,RCSdB(:,1)); xlabel('aspect angle (degrees)');

ylabel('relative RCS (dB)');

title('RCS vs. Aspect Angle, Many-Scatterer Case')

axis([0,360,10\*log10(N\*z)-30,10\*log10(N\*z)+10]);

% Compute a 100-bin histogram of the data for each target;

% plot the theoretical exponential distribution as well

%

density = zeros(100,Nt);

centers = [];

for r = 1:Nt

if (isempty(centers))

[count,centers]=hist(RCS(:,r),100);

else

[count,centers]=hist(RCS(:,r),centers);

end

bin\_size = centers(2)-centers(1);

area = sum(count)\*bin\_size;

density(:,r) = count/area;

% plot the histogram of the data for each target, and an overlaid

% exponential for each target, based only on that target's data

figure(3);

bar(centers,density(:,r),1);

hold on

msig=mean(RCS(:,r)); % compute mean of current linear-scale RCS data

exp\_pdf = exp(-centers/msig)/msig; % theoretical pdf on same x-axis values

plot(centers,exp\_pdf,'r','LineWidth',2)

xlabel('radar cross section (m^2)')

ylabel('relative probability')

title('PDF of RCS vs. Aspect, Many-Scatterer Case')

hold off

pause(0.5)

end

% Now average the separate histograms and plot with an overlaid exponential

% based on the mean of all the data for all of the targets

figure(4);

bar(centers,mean(density,2),1);

hold on

msig=mean(RCS(:)); % compute mean of all linear-scale RCS data

exp\_pdf = exp(-centers/msig)/msig; % theoretical pdf on same x-axis values

plot(centers,exp\_pdf,'r','LineWidth',2)

xlabel('radar cross section (m^2)')

ylabel('relative probability')

title('PDF of RCS vs. Aspect, Many-Scatterer Case, All Targets')

hold off

% As a sanity check, do a single histogram of all of the RCS data for all

% targets, using the sam bin boundaries as above. This should match figure

% 4 exactly.

figure(5)

[count,centers]=hist(RCS(:),centers);

bin\_size = centers(2)-centers(1);

area = sum(count)\*bin\_size;

density\_all = count/area;

bar(centers,density\_all,1);

hold on

plot(centers,exp\_pdf,'r','LineWidth',2)

xlabel('radar cross section (m^2)')

ylabel('relative probability')

title('PDF of RCS vs. Aspect, Many-Scatterer Case, All Targets')

hold off

% Now do the variation with frequency

% Input number of angles and nominal range and frequency

% Mf=input('Enter # of frequencies: ');

% f1=input('Enter start frequency (Hz): ');

Mf = 300;

f1 = 10e9;

df = 3e8/2/10;

% f2=input(['Enter freq step (Hz) (minimum=',num2str(df)/1e6,' MHz): ']);

f2 = 15e6;

fv = f1+(0:Mf-1)\*f2;

% Loop over radar frequencies

echof=zeros(Mf,Nt);

for k=1:Mf

% Loop over targets and individual point scatterers

% each column of 'echof' is a different target

for r = 1:Nt

for p=1:N

phasor=z\*exp(j\*4\*pi\*fv(k)\*norm([x(p,r)-R,y(p,r)])/3e8);

echof(k,r)=echof(k,r)+phasor;

end

end

end

% The magnitude-squared of the total complex echo

% is the RCS to within a constant

RCSf=abs(echof).^2;

RCSfdB=10\*log10(RCSf);

% Plot the dB data for the first target as an example

figure(6)

plot(fv,RCSfdB(:,1)); xlabel('RF frequency (Hz)');

ylabel('relative RCS (dB)');

title('RCS vs. RF Frequency, Many-Scatterer Case')

axis([0,360,10\*log10(N\*z)-30,10\*log10(N\*z)+10]);

% Compute a 100-bin histogram of the data for each target;

% plot the theoretical exponential distribution as well

%

densityf = zeros(100,Nt);

centersf = [];

for r = 1:Nt

if (isempty(centersf))

[countf,centersf]=hist(RCSf(:,r),100);

else

[countf,centersf]=hist(RCSf(:,r),centersf);

end

bin\_sizef = centersf(2)-centersf(1);

areaf = sum(countf)\*bin\_sizef;

densityf(:,r) = countf/areaf;

% plot the histogram of the data for the first target, and an overlaid

% exponential for that target, based only on that target's data

figure(7);

bar(centersf,densityf(:,r),1);

hold on

msigf=mean(RCSf(:,r)); % compute mean of current linear-scale RCS data

exp\_pdf = exp(-centersf/msigf)/msigf; % theoretical pdf on same x-axis values

plot(centersf,exp\_pdf,'r','LineWidth',2)

xlabel('radar cross section (m^2)')

ylabel('relative probability')

title('PDF of RCS vs. RF, Many-Scatterer Case, 1 Target')

hold off

pause(0.5)

end

% Now average the separate histograms and plot with an overlaid exponential

% based on the mean of all the data for all of the targets

figure(8);

bar(centersf,mean(densityf,2),1);

hold on

msigf=mean(RCSf(:)); % compute mean of all linear-scale RCS data

exp\_pdf = exp(-centersf/msigf)/msigf; % theoretical pdf on same x-axis values

plot(centersf,exp\_pdf,'r','LineWidth',2)

xlabel('radar cross section (m^2)')

ylabel('relative probability')

title('PDF of RCS vs. RF, Many-Scatterer Case, All Targets')

hold off

% As a sanity check, do a single histogram of all of the RCS data for all

% targets, using the sam bin boundaries as above. This should match figure

% 8 exactly.

figure(9)

[countf,centersf]=hist(RCSf(:),centersf);

bin\_sizef = centersf(2)-centersf(1);

areaf = sum(countf)\*bin\_sizef;

density\_allf = countf/areaf;

bar(centersf,density\_allf,1);

hold on

plot(centersf,exp\_pdf,'r','LineWidth',2)

xlabel('radar cross section (m^2)')

ylabel('relative probability')

title('PDF of RCS vs. RF, Many-Scatterer Case, All Targets')

hold off

Listing of xcorrRCS10\_project.m

% xcorrRCS10\_project

% same as RCS\_project but for computing correlation around zero degrees

%

% Mark Richards, September 2006

%

% Updated for averaging over multiple target, September 2010

clear all

close all

Nt=10; % # of targets to average over

% Set number and locations of scatterers. They are uniformly

% distributed within a box 5 m by 10 m centered at the origin.

% N=input('Enter number of scatterers to use: ');

N=50;

% each column of 'x' or 'y' are the scatterer corrdinates for a single

% target

y=5\*rand(N,Nt)-2.5; x=10\*rand(N,Nt)-5;

% Amplitudes are fixed = 1

z=1;

% Input number of angles and nominal range and frequency

% thetamax=input('Enter +/- range of angles (degrees): ');

% Dtheta = input('Enter angle increment (degrees): ');

% M = round(thetamax/Dtheta);

% R=input('Enter nominal range (m): ');

% f=input('Enter frequency (Hz): ');

thetamax = 3; Dtheta = 0.02; R = 10e3; f = 10e9; M = round(thetamax/Dtheta);

% Loop over aspect angle to do complex voltage estimates

q=zeros(2\*M+1,1); echo=zeros(2\*M+1,Nt);

% Loop over radar-target aspect angles

for k=1:2\*M+1

q(k)=(pi/180)\*((k-M-1)\*Dtheta); % current aspect angle

% Loop over targets and individual point scatterers

% each column of 'echo' is a different target

for r = 1:Nt

for p=1:N

phasor=z\*exp(j\*4\*pi\*f\*norm([x(p,r)-R\*cos(q(k)),y(p,r)-R\*sin(q(k))])/3e8);

echo(k,r)=echo(k,r)+phasor;

end

end

end

% autocorr of the complex data for each target

s = zeros(4\*M+1,Nt);

lag = Dtheta\*(-2\*M:+2\*M);

for r = 1:Nt

s(:,r) = xcorr(echo(:,r));

% plot autocorr of each target with markers

figure(1)

plot(lag,abs(s(:,r))/max(abs(s(:,r)))); xlabel('aspect angle (degrees)');

ylabel('normalized magnitude of autocorrelation');

title('Autocorrelation, 1 Target')

grid;

% plot theoretical markers

thetacorr = 3e8/2/5/f\*180/pi;

hold on

plot([-thetacorr,-thetacorr],[0,1],'-r');

plot([thetacorr,thetacorr],[0,1],'-r');

hold off

pause(0.5)

end

% Now do the average complex autocorrelation and plot and mark its

% magnitude

s\_avg = mean(s,2);

figure(2)

plot(lag,abs(s\_avg)/max(abs(s\_avg))); xlabel('aspect angle (degrees)');

ylabel('normalized magnitude of autocorrelation');

title('Autocorrelation, 10 Targets')

grid;

% plot theoretical markers

thetacorr = 3e8/2/5/f\*180/pi;

hold on

plot([-thetacorr,-thetacorr],[0,1],'-r');

plot([thetacorr,thetacorr],[0,1],'-r');

hold off

pause

% Now do the autocorrelation with frequency

% fmax = input('Enter +/- range of freequencies: ')

% df = input('Enter frequency step: ')

fmax = 30e6;

df = 1e6;

Mf = round(fmax/df);

% Loop over radar frequencies

fx = zeros(2\*Mf+1,1);

echof=zeros(2\*Mf+1,Nt);

for k=1:2\*Mf+1;

fx(k) = (k-Mf-1)\*df;

% Loop over targets and individual point scatterers

% each column of 'echof' is a different target

for r = 1:Nt

for p=1:N

phasor=z\*exp(j\*4\*pi\*fx(k)\*norm([x(p,r)-R,y(p,r)])/3e8);

echof(k,r)=echof(k,r)+phasor;

end

end

end

% autocorr of the complex data for each target

sf = zeros(4\*Mf+1,Nt);

lagf = df\*(-2\*Mf:+2\*Mf);

for r = 1:Nt

sf(:,r) = xcorr(echof(:,r));

% plot autocorr of each target with markers

figure(3)

plot(lagf/1e6,abs(sf(:,r))/max(abs(sf(:,r)))); xlabel('RF Frequency (MHz)');

ylabel('normalized magnitude of autocorrelation');

title('Autocorrelation, 1 Target')

grid;

% plot theoretical markers

fcorr = 3e8/2/10;

hold on

plot([-fcorr,-fcorr]/1e6,[0,1],'-r');

plot([fcorr,fcorr]/1e6,[0,1],'-r');

hold off

pause(0.5)

end

% Now do the average complex autocorrelation and plot and mark its

% magnitude

sf\_avg = mean(sf,2);

figure(4)

plot(lagf/1e6,abs(sf\_avg)/max(abs(sf\_avg))); xlabel('RF Frequency (MHz)');

ylabel('normalized magnitude of autocorrelation');

title('Autocorrelation, 10 Targets')

grid;

% plot theoretical markers

fcorr = 3e8/2/10;

hold on

plot([-fcorr,-fcorr]/1e6,[0,1],'-r');

plot([fcorr,fcorr]/1e6,[0,1],'-r');

hold off